Enacting Core Practices of Effective Mathematics Pedagogy with GeoGebra

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This research study was conducted to identify core practices of enacting effective mathematics pedagogy with GeoGebra, a software application for teaching and learning mathematics. Eleven Ghanaian in-service mathematics teachers were engaged in a twelve-month professional development programme where they developed GeoGebra-based mathematics lessons, which they taught to their peers and subsequently to students in the classroom. The teachers' actions and views about using GeoGebra to enact mathematics lessons were examined with the aim to specify the core practices of effective mathematics pedagogy. Data was collected through interviews, focus group discussions, lesson plans, and lesson observations. The results provided evidence that teachers were able to enact core practices, to different degrees, within five central themes of effective mathematics pedagogy: creating a mathematical setting, providing worthwhile mathematical tasks, orchestrating mathematical discussions, making mathematical connections, and assessing students' learning. Further analysis of the data provided evidence for theorising 31 core practices across these central themes of effective mathematics pedagogy. Following their engagement in the professional development programme, the teachers enacted these practices to greater or lesser extent. However, it was problematic for most teachers to effectively engage their students in a deep mathematical discussion. The findings from this study have implications for high school mathematics curriculum, effective mathematics pedagogy literature, and professional development for technology integration in teaching and learning.

Keywords · Effective mathematics pedagogy · GeoGebra · professional development. technology integration · teachers' TPCK

Introduction

Mathematics education literature (e.g., Calder et al., 2006; Jupri et al., 2015) and curriculum documents (e.g. Ministry of Education, Science and Sports, [MOESS], 2007) consistently accentuate the need to use technology to provoke students' thinking, exploration, and generalisation of mathematical concepts. For instance, the mathematics teachers in Ghana are expected to assist students to use computer applications such as spreadsheets and GeoGebra to explore problem-solving tasks (MOESS, 2007). The motivation for this mandate stems from the fact that these tools provide an alternative instructional approach which is consistent with the work of socio-cognitive theory (Geiger et al., 2012). This theory places the process of acquiring mathematical knowledge within the sociocultural and interactional settings (Vygotsky, 1978). For example, Springer et al. (1999) noted that when students are offered the opportunity "to discuss,

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debate, and present their own and hear one another's perspectives, … cognitive conflicts will arise, inadequate reasoning will be exposed, and enriched understanding will emerge" (p. 25).

Enhancing effective mathematics pedagogy using technology persists as a challenge for educators (Artigue, 2002; Davies, 2011). Despite extensive efforts by numerous scholars (e.g. Anthony & Walshaw, 2007; Smith, 1999) to develop models for evaluating effective pedagogy, the descriptions in most of these models are often generic and underspecified (Jacobs & Spangler, 2017). Another often articulated issue is the teachers' professional competence in using technology to engage students in shared dialogue when constructing mathematical concepts. Artigue (2002) identified that, while it is easy to see the pragmatic value of technology-assisted instruction (students preoccupied with using technology to find answers), it may be hard to spot the epistemic value of using technology in the classroom (students preoccupied with the thinking that leads to the solution) because teachers need to have sufficient pedagogical knowledge related to the use of technology.

For teachers to trade off the complexities involved in bridging the disjuncture between the pragmatic and epistemic values of the adoption of technology, this paper argues for (i) a process of professional development to enhance teachers' technological and pedagogical content knowledge (TPACK) (Mishra & Koehler, 2006) and (ii) strategic identification of the core practices of mathematics pedagogy that are likely to improve teaching and learning within a particular context (Jacobs & Spangler, 2017) such as GeoGebra. GeoGebra is a technological tool which covers mathematical content such as algebra, calculus, statistics, vectors, and geometry. This tool has gained popularity in mathematics education because it is easily accessible, user-friendly, and cost-effective for technology integration professional development (Hohenwarter et al., 2009). Thus, the central focus of this paper is to specify the core practices of effective mathematics pedagogy by identifying teachers' actions and views about the use of GeoGebra, following their engagement in the professional development programme.
The research question formulated to guide the study was:

What are the specific core practices of effective mathematics pedagogy in a learning environment that makes use of technology?

The move towards the specification of the core practices of effective mathematics pedagogy is an attempt to extend our understanding of the affordances and constraints of using technology to enact interactive mathematics instruction. Also, the findings may provide possibilities for consideration for improving the structure and content of professional development for teachers involving the use of technology in the mathematics classroom.

Conceptualising Effective Mathematics Pedagogy

Several terms have been used to connote effective pedagogy: standard authentic instruction (Newmann & Wehlage, 1993), professional standards for teaching mathematics (Martin & Speer, 2009), and active learning of mathematics (Smith, 1999). The common notion among these models of effective pedagogy is the creation of a learning environment which makes both teacher and students participative in constructing mathematical concepts.

In this paper, effective pedagogy is conceptualised to align with Anthony and Walshaw's (2007) perspectives of mathematics teaching and learning. Drawing on the mathematical proficiency framework of Kilpatrick et al. (2001), Anthony and Walshaw described effective mathematics teaching as pedagogical approaches that engage learners to achieve desired learning outcomes. The desirable learning outcomes espoused by these authors encompass students' ability to demonstrate the five strands of interrelated mathematical proficiency: conceptual

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understanding (comprehension of mathematical concepts, operation, and relation), procedural fluency (skills in carrying out procedures flexibly, accurately, efficiently, and appropriately), strategic competence (ability to formulate, represent, and solve mathematical problems), adaptive reasoning (capacity for logical thought, reflection, explanation, and justification), and productive disposition (habitual inclination to seeing mathematics as sensible and worthwhile, coupled with a belief in diligence and one's own efficacy) (Kilpatrick, et al., 2001).

Anthony and Walshaw (2007) suggested 10 principles of pedagogical practices that facilitate learning for diverse learners. These principles are consistent with the National Council of Teachers of Mathematics [NCTM] (2007) key practices of enacting quality mathematics instruction (Table 1). Both models have an explicit focus on student-centred learning, where the mathematics classroom is considered as a social space that includes students, the teacher, the subject of learning, and engaging activities (Vygotsky, 1978; Middleton et al., 2017).

Gervasoni et al. (2012) identified that creating a powerful learning environment and selecting rich mathematical tasks are among other factors that promote successful mathematics learning. Anthony and Walshaw (2009) noted that assessing students' mathematics understanding through classroom discourse (mathematical communication) could promote their adaptive reasoning where they extend their mathematical knowledge to unfamiliar situations. Also, choosing a worthwhile mathematical task (Anthony & Walshaw, 2007, 2009) or using a problem-oriented task in the classroom (NCTM, 2007) enhances students' productive dispositions where they develop inclination towards the worthwhileness of mathematics (Kilpatrick, et al., 2001).

From the above discussion, the themes of creating a mathematical setting, selecting worthwhile mathematical tasks, mathematical discussions, mathematical connections, and assessment of students' learning are deemed central to effective mathematics pedagogy. Also, the teacher requires the knowledge of technology, pedagogy, and mathematics content (TPCK) to be productive in using technology in the classroom (Mishra & Koehler, 2006).

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Table 1

Elements of effective mathematics pedagogy

Anthony & Walshaw (2007)	NCTM (2007)
An ethic of care - creating a classroom community that promotes the needs of individual students Arranging for learning Building on students' thinking Worthwhile mathematical tasks	• Choosing "good" problems that: - Exploring important mathematical concepts, - Providing students the chance to solidify and extend their knowledge.
Making connections	• Encouraging students to explore multiple solutions. • Encouraging students to think more deeply about: - the problems they are solving - making connections with other ideas within mathematics
Assessment for learning	• Assessing questioning techniques to facilitate students' learning and reasoning.
Mathematical communication	• Assessing students' understanding by: - listening to discussions - asking students to justify their responses • Creating a variety of opportunities such as - group work and - class discussions for students to communicate mathematically.
Mathematical language	• Model appropriate mathematical language and strategies for solving challenging mathematical problem.
Tools and representations Teacher knowledge and learning	• Use multiple representations to foster a variety of mathematical perspectives.

Creating a Mathematical Setting

The first category of effective mathematics pedagogy pertains to setting up the environment to induct students into mathematics learning. It involves the actions the teachers take before, during, and after the lesson. The creation of a mathematical setting is conceptualised to encompass the skills and knowledge of the teacher to set up the learning environment to sustain the interest and attention of the students. The concept of an ethic of care reflects the considerations the teachers take to ensure that students have opportunities to construct positive relationships through interactions and dialogue (Ingram, 2013). Franke et al. (2007) reiterated that particular norms for classroom communities are important to ensure that value is placed on: ideas and methods through which an answer is obtained, autonomy in choosing and sharing mathematical ideas, appreciation of student's mistakes, and renegotiation of mathematical ideas.

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Worthwhile Mathematics Tasks

A task is worthwhile if the objectives and activities associated with the task engage students in exploring mathematical concepts, procedures, and/or relationships. A worthwhile mathematical task supports students to develop engagement skills such as perseverance, mathematical intimacy and integrity, independence, concentration, cooperation, and reflection (Ingram, 2013). An important aspect of effective pedagogy is the provision of an important mathematical task that offers opportunities for students to extend their mathematical knowledge and thinking and skills of problem-solving (Hiebert & Grouws, 2007).

Mathematical Discussions

Mathematical discussions involve creating opportunities for students to communicate their mathematical dispositions, conjectures and concepts (Stein et al., 2008). Stein, et al. (2008) identified five key practices crucial for orchestrating mathematical discussion. These included anticipating students' responses, monitoring individual or group work, selecting work to be presented for whole class discussion, sequencing the work presented by individuals or groups of students, and facilitating connection-making between students' responses. In this model of enacting mathematical discussion, the teacher needs to anticipate and set out strategies to address the possible responses the students are likely to provide to each sequence of activities included in the lesson plan.

Mathematical Connections

Mathematical connectivity refers to how the teacher engages the student to use an unrehearsed approach to generate multiple solutions to a task. The need to focus mathematics instruction more on problem-solving, applications, and higher-level thinking skills was recommended over three and a half decades ago (NCTM, 1980), and recent literature reiterates the relevance of including open-ended tasks to enhance students to make sense of the new mathematical concepts and skills (Jacobs & Spangler, 2017). According to Anthony and Walshaw (2007), effective teachers support students in creating connections between different ways of solving problems, between mathematical representations and topics, and between mathematics and everyday life.

Assessment of Students' Learning

Assessment in mathematics learning is a broad topic. Assessment is used to monitor students' mathematical learning, inform teachers' future instruction, update parents about their child's learning progress, determine the mathematical achievement of a country's students, and inform policy direction in education. In this paper, assessment is operationalised as the creation of a learning environment where formative feedback and feedforward from the teacher and students are promoted to monitor the progress of students' learning in a specific mathematical task. Effective teachers support their students' learning by: providing students with appropriate feedback about their thinking, encouraging learners to self- and peer-assess their solutions, and using students' thinking to sequence mathematics instruction (Martin & Speer, 2009).

Teachers' TPCK

TPCK is an extension of the pedagogical content knowledge (PCK) framework, introduced by Shulman (1986). With regards to the introduction of technology in mathematics education, Mishra and Koehler (2006) stressed the need for teachers to have integrated knowledge of

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technology, pedagogy, and content (TPCK). TPCK comprises what the teacher knows and believes about the nature of mathematics, what is important for students to learn, and how technology supports learning mathematics. These foundations of the teacher's knowledge and beliefs about teaching mathematics with technology serve as the basis for his or her decisions about classroom instruction (objective, strategies, assignments, curriculum and text, and evaluation of student learning). Getenet et al., (2014) suggested that access to technology resources as well as authentic, collaborative learning experiences can produce notable learning improvements for teachers to effectively use technology in their classroom practices.

Potential of GeoGebra

Many studies have demonstrated the efficacy of GeoGebra in teacher professional development for technology integration (Andresen & Misfeldt, 2010; Hudson, 2012; Prodromou et al., 2015). Andresen and Misfeldt (2010) gave an account of how GeoGebra was used to enhance the knowledge and skills of teachers towards the use of technology in secondary school mathematics. The authors observed, for instance, that the tessellation (repeated patterns that embody the concept of the polygon) material in GeoGebra initiated discourse towards effective practices in the mathematics classroom. Using the tessellation, the teachers asked more open-ended questions about how they could use it to support students' learning. The teachers at the end of the professional development programme became enthused about helping students to use ICT facilities to create mathematics artefacts and share them with their colleagues. Other studies have also demonstrated multiple functionality of GeoGebra in the mathematics classroom. The pre-service teachers in Bulut and Bulut's (2011) study were able to use GeoGebra to (i) create interactive web pages for students' mathematics learning, (ii) enact discovery-based learning where real-life examples were used to develop mathematical concepts and thinking, and (iii) create multiple representations to aid students' construction of mathematical ideas.

The above studies suggest that professional development, which is activity-driven (where a small group of teachers come together to explore, negotiate and design technology-based mathematics lesson) can provide opportunities for teachers to develop in-depth understanding of theories and practices needed for effective use of technology.

Research Design

The study adopted a case study research approach where eleven in-service mathematics teachers from a senior high school in Ghana voluntarily engaged in professional development for 12 months. The age of the teachers ranged between 20-45 years and their teaching experiences, 1-16 years. The teachers gave their informed consent for inclusion before they participated in the study. The study followed the ethical protocol approved by the Human Ethics Committee of the University of Otago (Reference Number: 17/014).

The professional development involved three phases: (i) collection of preliminary data, (ii) workshop training, and (iii) enactment of GeoGebra-based mathematics lessons in classrooms. The first phase lasted four weeks. Preliminary data about the teachers' demographic information and their experiences of current use of technology were collected to inform the subsequent phases of the professional development. This initial information was used to guide the professional development programme. The second phase lasted four months. Weekly meetings lasting up to three hours were organised to introduce the teachers to the basic algebraic and constructing tools in the GeoGebra window. They used these tools to explore pedagogical approaches of teaching mathematical concepts such as polygons, mensuration, and graphs of polynomial and

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trigonometric functions. A tutorial manual for the activities was given to each participant to enable them to practice at their own pace before the weekly meetings. After the introduction to the tools in GeoGebra, a five-day intensive training workshop was organised for the teachers during the school holidays in August 2017. Each session lasted for a maximum of six hours. The teachers designed and taught a GeoGebra-based mathematics lesson to their peers. They were engaged in discussion after each episode of peer teaching to critically reflect and share the best practices of using GeoGebra to enact mathematics lessons. In the third phase, the teachers were then encouraged to use GeoGebra in their classrooms. Interviews, focus group discussions, lesson plans and lesson observations were used for data collection. Before describing the procedure for data collection and analysis, a snapshot of the education system in Ghana and the research setting is presented.

The Education System in Ghana

The new educational reform in Ghana took effect in the 2019/2020 academic year. Per the reform, the education system is divided into two parts: basic and tertiary education. The basic education is free and compulsory and it lasts until 14 years (age 4-17). It is the minimum period of schooling required to develop basic literacy, numeracy, healthy living skills, digital skills, and problem-solving skills as well as "a sense of identity as creative, honest and responsible citizens" (Ministry of Education, 2019, p. i). The basic school has four components: Kindergarten (ages 4 to 5 years), primary school (ages 6 to 11 years), junior high school (ages 12 to14 years), and senior high school (ages 15 to 17 years). Before a child completes the junior high school, he/she takes an external examination called the Basic Education Certificate Examination (BECE). The child must obtain an aggregate threshold mark in the BECE to qualify him/her to enter senior high school. The senior high school is equivalent to high school as it is in many western countries. The Tertiary Education component comprises college and university education (ages 18 to 21 years). It is important to note that although an age bracket has been assigned to each level of education in Ghana, it does not necessarily restrict someone from entering any level. The age for someone to enter any level of education in Ghana is most often dependent on the person's socio-economic background. In most cases, however, it is expected that by age 21 one should have completed tertiary level education.

The Research Setting

The study was conducted within the context of senior high school mathematics curriculum in Ghana. The school had an enrolment of 1600 students, 33 classrooms for teaching, and a science laboratory. The school had an ICT laboratory with a projector, two desktop computers, four mini laptop computers, and a printer. By nature of the ICT laboratory in the school, the teachers were only able to use a laptop computer and a projector to conduct their lessons. GeoGebra was used as an explorative tool where lesson artefacts pre-developed by the teachers were projected on the screen to initiate demonstration, whole class discussion, and concept formation.

Interviews and Focus Group Discussions

The teachers' views about the use of technology to enact mathematics lessons were explored through audio-recorded interviews and focus group discussions. Each participating teacher was interviewed twice (lasting not more than 60 minutes), before and after the professional development. The teachers were engaged in multiple focus group discussions at the various phases of the professional development programme. The interviews and focus group discussion

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data were analysed inductively (Thomas, 2006). The audio-recorded interviews and discussions were organised and listened to repeatedly to enhance familiarisation with the data. The audio files were then transcribed. Constant reflection of the data was achieved through the use of third column analysis. Third column analysis involves writing the interpretation of the transcripts at the right-hand column. This interpretation was done repeatedly to make initial sense of codes (Ingram, 2011). Comment, text highlight colour, and font colour features in Microsoft Word were used to achieve this. The transcribed data were then imported into HyperRESEARCH 4.0.1. for detailed coding and analysis. The bar graph and cluster features in the HyperRESEARCH facilitated clustering the codes into themes.

After the first pass of the coding, two experienced mathematics educators reviewed the initial themes in relation to the data set. They supported the author to map links between codes by paying attention to the responses the teachers provided to each question for the purpose of refinement, recoding, and thematising (Braun & Clarke, 2013), until more than 80% agreement on each theme was reached. The final themes were imported to Microsoft word for organisation (Table 2). Pseudonyms were used for all names.

Table 2

Examples of coding used to analyse teachers' view about the use of technology in teaching and learning

Sample Sub-themes	Sample Codes	Sample quotations
Pedagogical tool	Explorative instruction Socio-cognitive learning Faciliatory instruction	Using the technology assists students to learn cooperatively because when a student presses [the computer] and he or she does not get the answer he will quickly go to a friend for a support. Through that they will be sharing ideas. … My role only becomes a facilitator rather than imposing formulas and other things on the students (Peter).
	Mathematical connection	If we are able to use technology in our instruction, students can easily translate whatever they learnt in school in solving problem outside the school (Martey).
	Visual representation	With the projector and laptop, you can show whatever you have for the students to have a feel or to see it real. That one will help them see it clearer and they will understand it better. They will get it real than the abstract teaching (Michael).

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Assessment practice tool	Monitor students learning Prompt feedback	In the classroom where there is internet connectivity, the teacher can easily google to get a pictorial view of the question for the students. So, from introduction down to the assessment, they are well aware of what is happening (Joshua).
	Continuous assessment	The teacher can have his or her continuous assessment sheet on the computer. It can assist you to add the figures easily. You just have to enter the figure, and it gives you the total. Unlike sitting down and doing it manually, trying to calculate everything (Bernard).

Lesson Plans and Lesson Observations

The lesson plans and lesson observations were used as evidence to evaluate the progress of the development of the teachers' knowledge and use of technology in mathematics teaching. The key moments of the teachers' actions were recorded in the field notes (logbook). The lesson plans and transcribed lesson episodes were imported into HyperRESEARCH. The data were deductively analysed using the effective pedagogy framework proposed for the study. The core practices that became common were regrouped and interpreted under the themes: creating a mathematical setting, worthwhile mathematical tasks, mathematical discussions, mathematical connections, and assessment of students' learning. It is important to acknowledge that some of the teachers' actions were multiple coded for different themes because there were overlap of the actions of the teachers for different themes. For example, the excerpt from Sammy's lesson was coded creating a mathematical setting as well as a worthwhile mathematical task:

> Observe the polygons shown on the GeoGebra window and record your observations in the table below. From the table in (i) write down the number of triangles in a given polygon in terms of n, (ii) write down the formula for the sum of interior angles of a polygon (S) in terms of n (Sammy).

The excerpt indicates that the teacher has created a mathematical setting because it served as spark for engaging students to the task. In other words, the teacher used the animated polygons in the GeoGebra window to catch the attention of the students. The excerpt also indicates it was a worthwhile mathematical task because it engaged students to use certain mathematical proficiencies such as observation, recording and deduction of mathematical concepts. This overlap illustrates the non-linearity and complexity of teachers' practices in the classroom.

Results

The study identified 31 core practices when the teachers used GeoGebra to enact mathematics lessons. These core practices are distributed over the five central themes of effective mathematics pedagogy adapted for the study: creating a mathematical setting, providing a worthwhile mathematical task, engaging in mathematical discussion, making mathematical connections, and assessing students' learning.

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Creating a mathematical setting

The teachers demonstrated nine key practices in setting up an effective environment for mathematics learning. These include creation of pre-planned documents (lesson plan, students' worksheet and GeoGebra artefacts), equipment setups, shared instructional objectives, clear instruction about the task on the worksheet, linking prior knowledge to new learning, using pictures of real-life scenarios to initiate mathematical dialogue, explanation of terminologies, addressing existing misconceptions, and attention to individual learning needs. The analysis of the lesson plans revealed that the teachers prepared high-quality lesson plans that made explorative use of GeoGebra. The instructional objectives they included in their lesson plans aligned with the targeted grade level and topics indicated in the mathematics curriculum. Also, all the teachers included introductory activities which inducted the students into the lesson. Five teachers included expected solutions/responses to the tasks the students performed in their lesson plan documents. The teachers projected the expected solutions/responses on the screen to aid students to compare their answers. This enabled the teachers to elaborate the concepts they wanted the students to develop. Also, the worksheets engaged the students in hands-on activities. The GeoGebra artefacts included texts, diagrams, shapes, graphs, and animations which were designed in the GeoGebra window, to provide a material for presentation and exploration of mathematical concepts.

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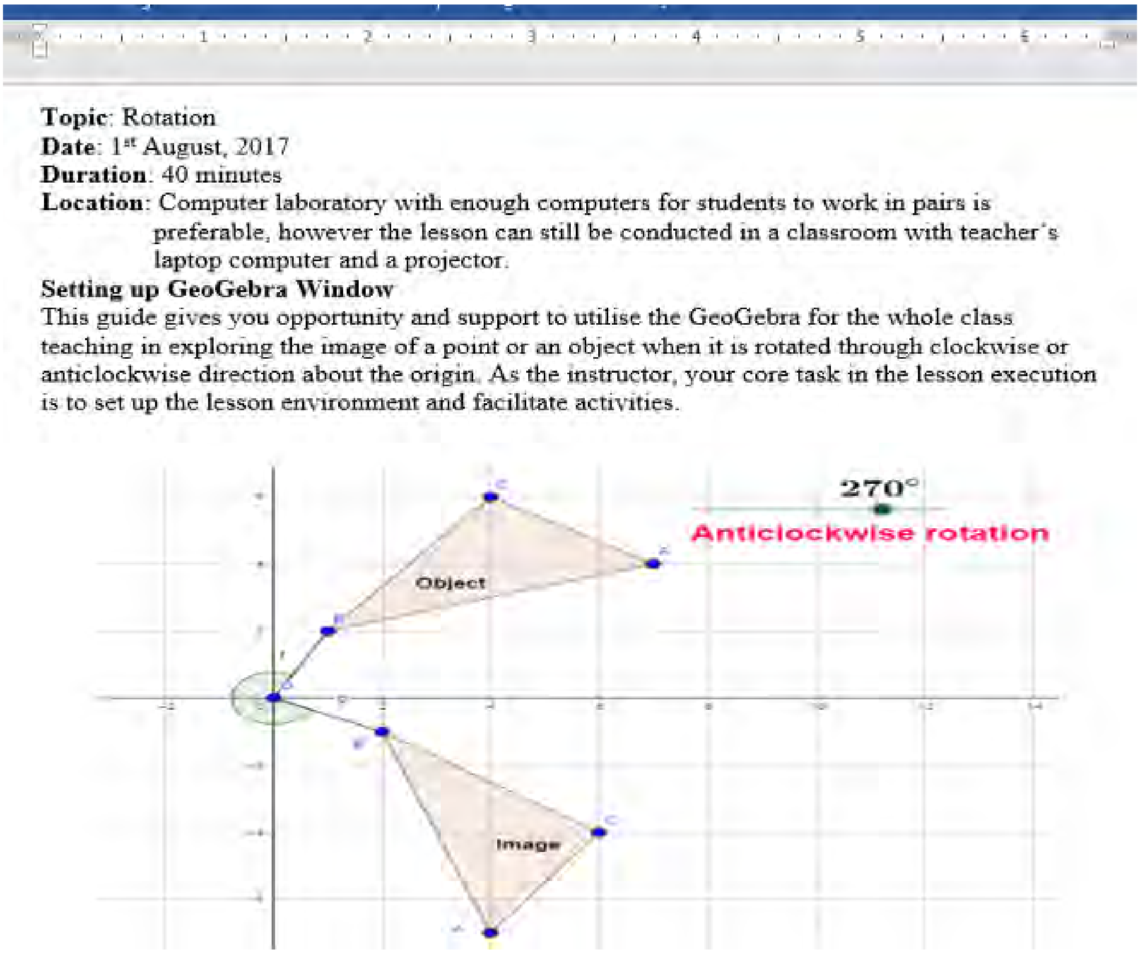
In Figure 1 for example, Cynthia was able to set out the GeoGebra window for her lesson on rotation of objects. Before the lesson, she stated that often students missed the formula which made them unsuccessful to locate the appropriate quadrant for an object when it is rotated. She noted from her lesson that the dynamic movement of the object in the GeoGebra window facilitated her students to figure out which quadrant the image of the object would be in without doing any computation or memorisation of formula.



Topic: Rotation
Date: 1st August, 2017
Duration: 40 minutes
Location: Computer laboratory with enough computers for students to work in pairs is preferable, however the lesson can still be conducted in a classroom with teacher's laptop computer and a projector.
Setting up GeoGebra Window
This guide gives you opportunity and support to utilise the GeoGebra for the whole class teaching in exploring the image of a point or an object when it is rotated through clockwise or anticlockwise direction about the origin. As the instructor, your core task in the lesson execution is to set up the lesson environment and facilitate activities.

Figure 1. Setting up GeoGebra window for rotation of object (Cynthia).

She also stated that setting up the GeoGebra window in advance made her pedagogically oriented by predetermining how the technology could be used in her context (small group and whole classroom teaching), the role she would play (facilitating), and the expectation from her students (exploring mathematics concepts).

There was evidence that overall, the teachers hooked students' interest in the lessons by: (i) providing explicit instruction on the worksheets for the students to work with, (ii) articulating

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the instructional objectives of the lesson, (iii) reviewing students' existing knowledge, (iv) using real-life scenarios to address existing misconception, and (v) explaining terminologies involved in the concept. Each teacher used at least one of these strategies during the introduction stage of their lessons. It is important to acknowledge that the teachers repeated these strategies as the lessons unfolded, depending on the nature of the topic they taught, the background of their students, and teaching style and experience of the teacher. For example, in the following excerpt (from the lesson episode), Michael articulated the expectation of the lesson was to draw the attention of the students to what they were going to learn:

> In this lesson, you will be taken through activities that will help you explore the concept of trigonometric ratios: sine, cosine and tangent. It is expected that at the end of the lesson you will be able to apply trigonometric ratios to calculate distance and height of a given triangle (Michael).

From Joshua's perspective (Figure 4), contextualising technology to meet the needs of students is key in promoting an effective learning environment. He indicated in the post interview how the technology could be used to support the students. He not only used the technology to address individual learning differences, but also to resolve students' misconceptions in learning distance-time graphs:

> If it is more visual, it helps the weaker students to visualise the concept than without it. This makes all of them move forward. In my lesson, they all saw the car moving and when it stopped too, they all saw it. This makes it easier for them to make meaning out of it. Initially, some of them thought when the car stops, its time too stops. So, the concept of the distance covered by the car in relation to time was difficult for them to grasp especially when the car is at rest or return to its original position. (Joshua).

In Gideon's case, he downloaded real-life scenarios of a quadratic equation (Figures 2a and 2b) and used them to elicit responses from the students on the topic. Gideon sequenced his questions with the hope that the students would use terms such as parabola, or trajectory, or quadratic curve. However, the students were not able to generate these terms automatically, they only said the paths represent a curve. The use of the word 'curve' gave Gideon a good starting point for his lesson. He proceeded by saying such a curve is called a parabola or a trajectory. He then introduced the mathematical representation of the parabola, $y = ax^2 + bx + c$ to the students. The opportunity for the students to observe and talk about the path of the motorcyclist (Figure 2a) and that of the basketball (Figure 2b) initiated a mathematically fruitful activity where the students were less burdened with heavy abstraction of the concept of quadratic equations. For example, Gideon was able to support the students to build acceptable and formal terminologies from the common vocabulary (in this case, 'curve') they had used in describing the path of the motorcyclist.

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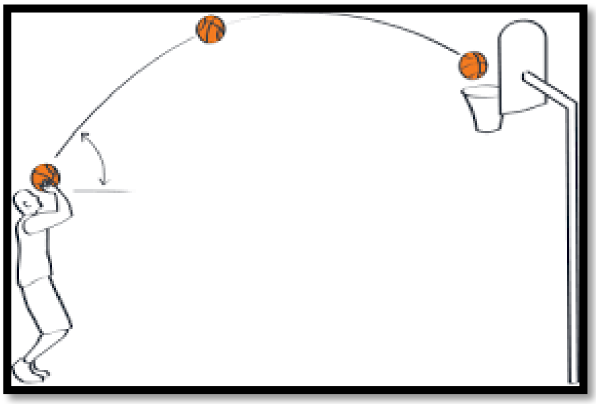
(a)



(b)

Figure 2. Illustration of real-life application of quadratic equation (Gideon). [1]

Worthwhile Mathematical Tasks

The teachers demonstrated five key practices in providing worthwhile mathematical tasks to their students: visualising mathematical concepts, recording, calculating, predicting and constructing new ideas. Nine teachers used GeoGebra to engage students in geometric thinking. In the following excerpt (from the lesson plan) for instance, Bernard (Figure 3) provided guided exploratory learning where students made geometric deductions from a sequence of activities including recording, observing patterns, drawing, guessing, calculation and conjecturing. The affordance of the animation in GeoGebra facilitated the students to identify the connection between the area of a rectangle and the curved surface of the cylinder (Figure 3). The dynamic development of the cylinder from its nets in the GeoGebra window helped Bernard to assist the students to realise that the total surface area of a cylinder is the sum of the area of the two circles plus the area of the curved surface ($2\pi r^2 + 2\pi rh$).

> Based on your observation, write the formula connecting surface area (SA), curved surface area, circular ends area and height (h) of the cylinder and the cone (Bernard).

[1] Source
(a) https://www.ridinginthezone.com/wp-content/uploads/2014/10/downhill-curve.jpg
(b) https://www.thinglink.com/scene/725731041792229376

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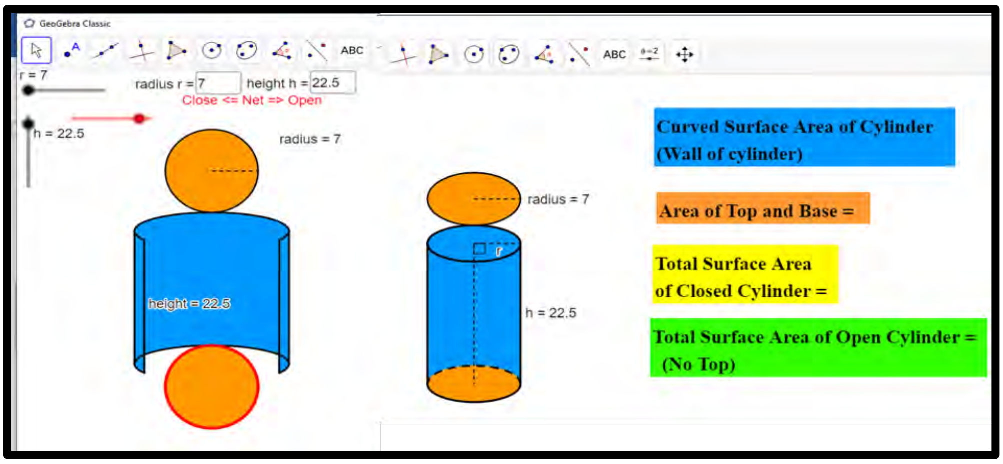

Figure 3. Surface area of a cylinder (Bernard).[2]

Similarly, in Joshua's lesson, he adapted an existing lesson from the GeoGebra online community platform to support his students to draw distance-time graphs for given scenarios. The students subsequently used the graph they had drawn to calculate distance travelled, total time taken, and the average speed of the moving object (Figure 4).

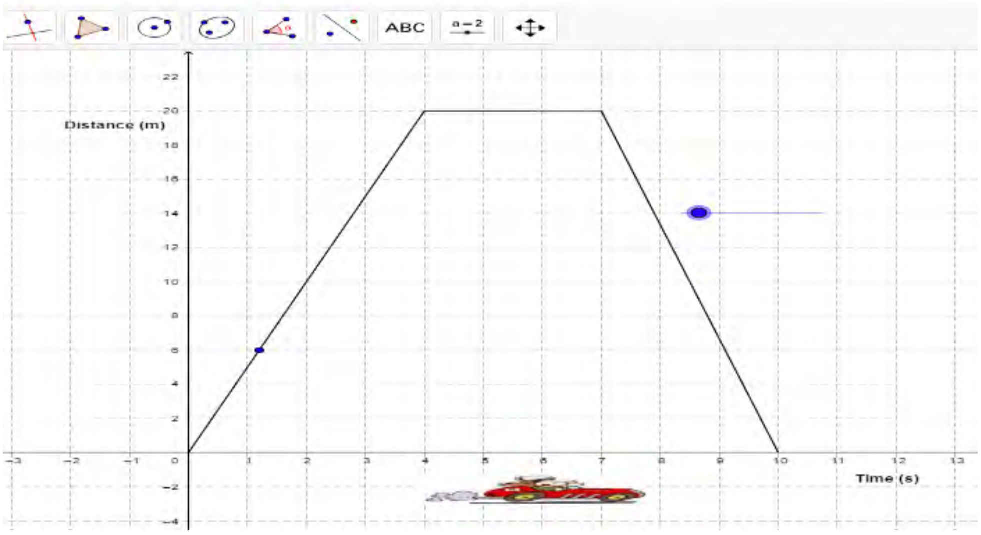

Figure 4. Travel Time Graph (Joshua).[3]

From the interaction that took place, Joshua provided a good start by asking students to talk about multiple travel graphs presented in GeoGebra. This made the students focus on key mathematical

[2] Source: https://www.geogebra.org/m/wknM5zxv

[3] Source: https://www.geogebra.org/m/p3xCqUtZ

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ideas for drawing travel graphs. For example, the animation in the GeoGebra window showed the movement of the car and its corresponding graph simultaneously. This supported the students to realise that positive slope (⋰) indicates that the car is moving forward, horizontal line (⋯) indicates that the car is at rest, and negative slope (⋱) indicates that the car is returning to the starting point. Making this connection was conceptually worthwhile to facilitate the students to draw and interpret their own graph when different scenarios were presented to them.

Mathematical Discussions

Engagement of students in mathematical discussions was a common feature across the lessons the teachers enacted. The results revealed six key practices of enacting effective mathematical discussion. These were consolidating ideas students had constructed through verbal and written responses, correcting misconceptions associated with the new concept, posing mathematical questions, group/individual presentation, revoicing, and giving students autonomy to apply the concept. The following excerpt is an illustrative example of how the teachers enacted these practices. The excerpt also shows some complexities associated with orchestrating effective mathematical discussion. In one of the activities in Martey's lesson, he guided his students in a whole class discussion to arrive at a rule for drawing different rectangles which have the same perimeter. He posed this question to the students: Draw three different rectangles such that each of them will have a perimeter 20 cm. He provided a square dot paper for the students to present their solution on it.

Martey:	Did you have any trick for doing it? I mean three different rectangles each having a perimeter of 20 cm.
Group1:	We were able to draw a rectangle with the dimensions 6 and 4.
Group 2:	Ours have a dimension: 2 and 8. We are still thinking.
Martey:	Good effort. Keep thinking.
Group 1:	We have gotten another one, the length is 7 and breadth 3. [Martey listed the dimensions the students provided on the board.]
Martey:	We now have 6 and 4, 7 and 3, and 8 and 2. I think you can have another one too. What do you think?
Group 1:	Then 9 and 1.
Martey:	When you look at this pair of numbers carefully, you will see some pattern.
S3:	The sum of the length and the breadth is 10.
Martey:	That is right. Why is it that the sum of the length and breadth is 10 and not any other number?
S8:	The perimeter is 20. So, when we divide 20 by 2, we have 10.
Martey:	We have to divide by 2 because the perimeter is given as $2(l + b)$. So, anytime you are given the perimeter and you want to determine the dimensions of different rectangles, you have to divide it by 2 first. Then you think about pairs of numbers that their sum will give you the result you obtained after the division.

In this excerpt, both the teacher and students seemed engaged. In line three, students acknowledged the struggling they were going through, but encouraged themselves by saying "We are thinking". Martey's inquiring attitude in line eight invited the students to develop ideas by formulating a rule for determining the dimension of a rectangle when its perimeter is given. He rewarded students and encouraged them to "keep thinking" when the task seemed harder for the students. The teacher asked further questions for students to explain how they got the answer (line 10). He revoiced and edited students' responses in the closing remarks (line 12).

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However, the conversation may have been richer if he had provided the opportunity for students to critique the solution from each group. It was observed that the kind of argument that went on was mainly within groups but not between groups. The groups were not subjected to public (whole class) scrutiny where divergent ideas could have evolved. The excerpt also suggests that the teacher had a predetermined solution to the question. In most cases the discussion ended prematurely as soon as the students provided the correct answer. The teacher did not invite any alternate solution from the students. Like other teachers in the study, he mostly selected groups they had pre-rehearsed the solution with for presentation and whole class discussion.

The following excerpt illustrates an interaction Michael had with his student on a task involving trigonometric ratios (Figure 5).

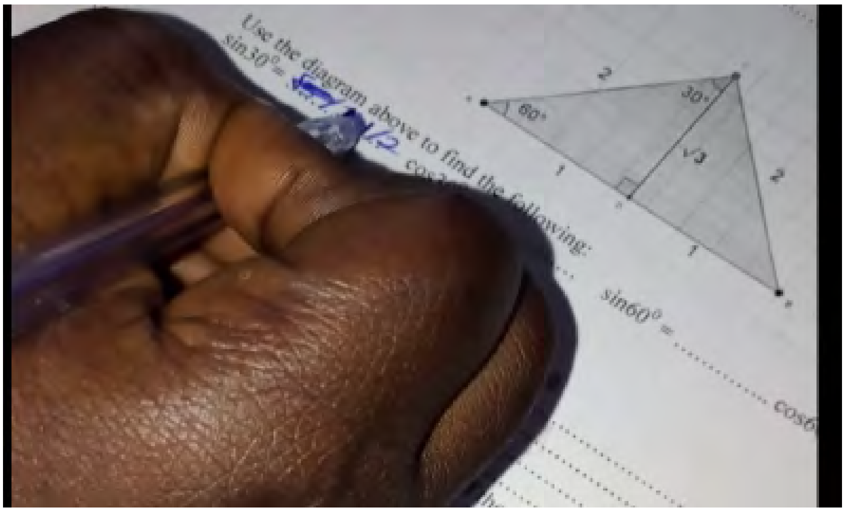

Figure 5. Sample solution on a task involving trigonometric ratios.

Michael:	What have you written?
Student 4:	sin 30^0
Michael:	What did you write for sin 30^0?
Student 4:	$\sqrt{3}$ over ...
Michael:	Let's look at the diagram. What is the value of $sin\theta$?
Student 4:	Opposite [side] over hypotenuse [side].
Michael:	Now let's look at sin 30^0. Where is the opposite [side]?
Student:	Here [pointing at 1]
Michael:	Good. Write it.
Student 4:	[She cancelled the one she had earlier written and wrote 1/…]
Michael:	Where is the hypotenuse [side]?
Student4:	Here [pointing at 2]
Michael:	What is [the value of] sin 30^0?
Student 4:	One over two [1/2].
Michael:	Good.

In this excerpt, Michael was giving attention to the individual student. From the conversation, Michael realised that the student had begun the solution wrongly by writing $sin30° = \sqrt{3}/?$ (line4). It was likely the student would complete the task incorrectly by writing either $sin30° =$

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$\sqrt{3}/1$ or $sin30° = \sqrt{3}/2$. Michael was not patient enough for the student to complete the task and he interrupted (line 5). Since it was the first time the student was working independently after the group work, it was possible she needed time to assimilate what she learned from the group members. She could have changed her solution or sought for assistance from her group members. Michael could have asked the student to explain her solution. This might have given him the opportunity to appreciate the misconception of the student and further use it to correct the other students who might have had a similar struggle. Though the student in the middle of the lesson knew that for a right-angled triangle, the value of $sin\theta$ is determined by dividing the length of the opposite side by the length of the hypotenuse side (line six), conceptually she was struggling to correctly identify the opposite and hypotenuse sides from the diagram. Though Michael eventually succeeded in helping the students to come up with the correct solution, it is likely that this particular student may not have fully understood the concept.

Mathematical Connections

Six key practices were apparent in the way the teachers enacted mathematical connection in their classrooms: reposing mathematical questions, extending concepts learnt to new contexts, using GeoGebra to emphasis real-life phenomena, reversed thinking, multiple solutions, and risk taking. Six teachers created scenarios that enabled students to link newly acquired concepts to real-life phenomena and vice versa. For example, Martey used a question which started with a solution to engage students in geometric thinking. The task he presented to the students was:

> Draw on the grid sheet three different rectangles such that each of them will have an area of 24 cm^2.

In this activity, he was able to immerse the students into reversed thinking where they used the solution to look for related mathematics concepts that could connect geometric drawing and algebraic symbols. This generated arguments, predictions, and formulations of mathematics concepts among the students. Students took risks by making correct and incorrect computations/decisions of drawing possible geometric figures before they arrived at the conclusion. Martey used the animated rectangle he generated in the GeoGebra window to clarify students' doubts and misconceptions during the whole class discussion (Figure 6).

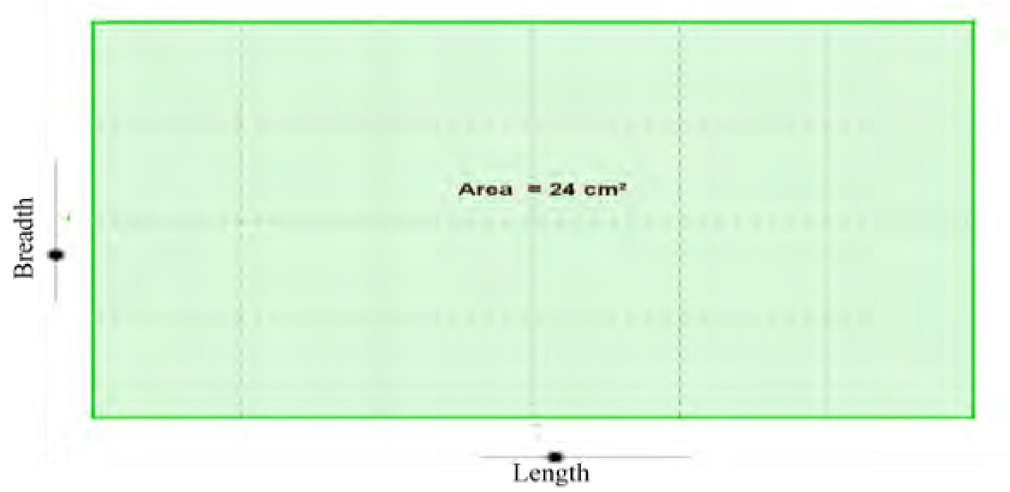

Figure 6. Area and perimeter of rectangle (Martey).

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In Bernard's lesson, he wanted his students to apply the concept of volume of cylinder they had learnt to calculate the volume of the metal sheet needed to make a pipe which height was 10 cm, internal radius 2 cm and external radius 2.4 cm (See Figure 7a).

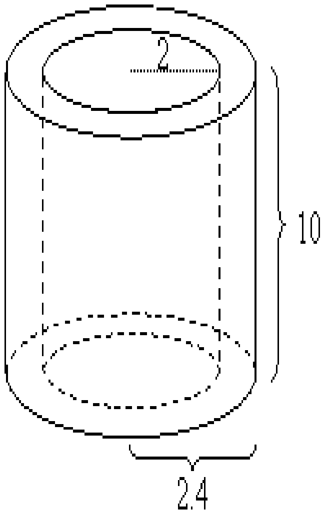

Figure 7a. Sample task on mensuration (Bernard).

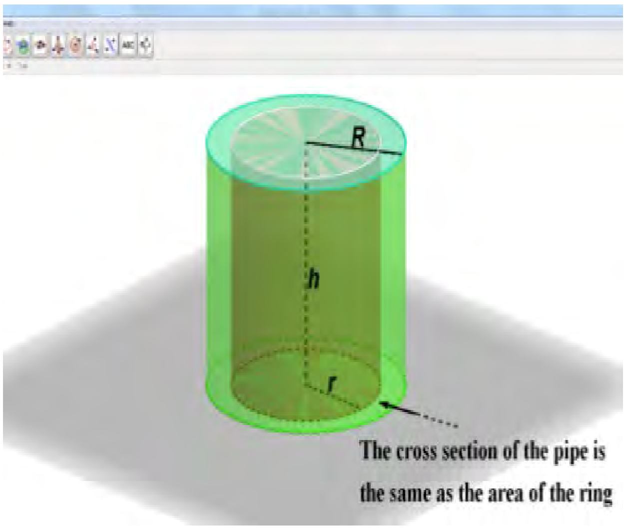

Figure 7b. Cylinder in the 3D graphics in GeoGebra (Bernard).

The task (figure 7a) challenged the students. At the first attempt, the students were not successful in this task, and very few of them made any progress. Those who attempted it performed a single calculation using the formula $\pi r^2 h$, but were unsure about which of the radii to use. Geometrically, they could not recognise that the pipe was hollow, and that they needed to calculate the area of the cross section of the pipe. When the teacher realised this limitation, he used GeoGebra (Figure 7b) to provide a dynamically pictorial hint which prompted the students to identify the geometric and algebraic relationship needed to solve the problem. The students visualised that to calculate the volume of the metal used to make the pipe, they had to calculate the cross section of the pipe, $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$ and further multiply the result by the height (h). On the other hand, they could calculate the volume of the bigger ($\pi R^2 h$) and the smaller ($\pi r^2 h$) cylinders separately and then determine the difference.

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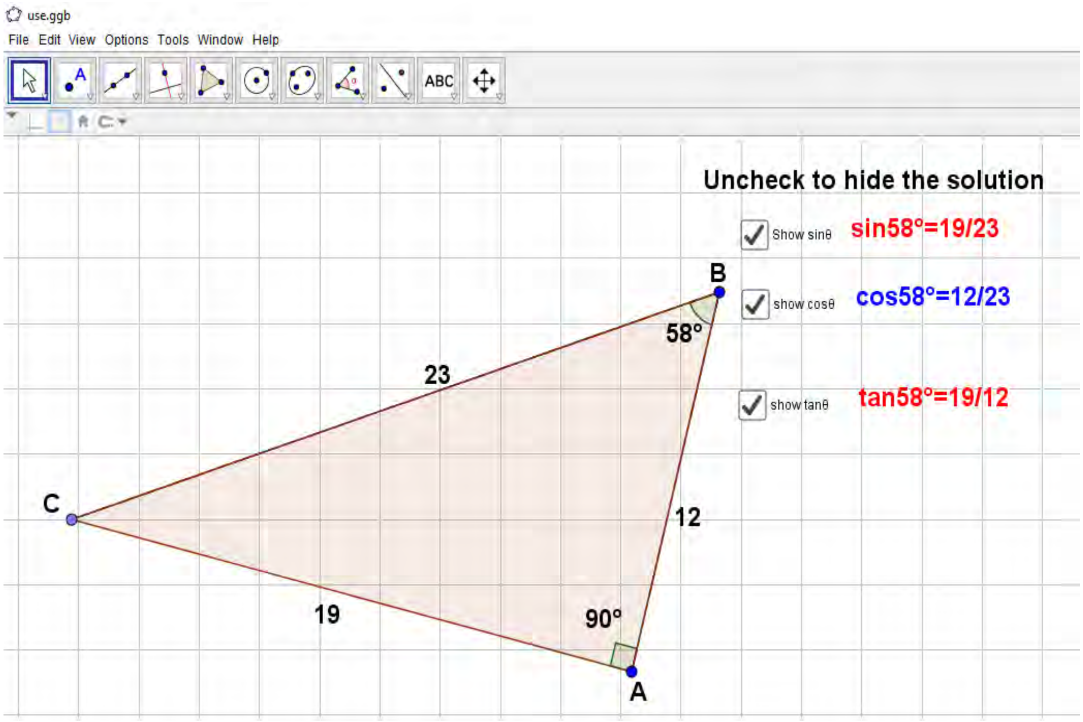
Assessment of Students' Learning

Five key practices were common in the way the teachers adopted GeoGebra and worksheets to assess their students' learning. The teachers reviewed and corrected students' errors, used GeoGebra to provide prompt feedback, provided remedial teaching for students who needed special attention, provided opportunity for students to share and reflect on their solutions. For example, Joshua, Bernard, Sammy, and Michael used the slider and checkbox features in GeoGebra to hide and unhide the solution they wanted the students to provide. The teachers unchecked the checkbox to show the solution after the students had performed the task to enable them to compare their answers for whole class discussion. In Michael's case, he used the affordance of the checkbox in GeoGebra to consolidate his students' knowledge and skills of applying appropriate trigonometric ratio (Figure 8).

Figure 8. Affordance of the checkbox in GeoGebra for assessment purposes (Michael).



This feature in GeoGebra supported the teachers to create multiple questions for the students within a short time. It also provided learners with immediate feedback about their solutions as well as a platform for sharing their thoughts about their own solutions.

There was evidence in the data that teachers provided advance information to prevent students from making possible errors. For example, in the excerpt below, Sammy invited the students into a conversation about the sum of the interior angles of a polygon. He anticipated possible errors the students could make in determining the numbers of triangles in a polygon and provided a caution to prevent the students from falling into that trap (line 8). Also, he knew some of the students could wrongly arrive on the deduction $S = 180° \times n - 2$ instead of $S = 180°(n -$

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2) (line 10). However, he allowed the students to commit that error which he used to elaborate on the appropriate way of writing the formula.

Sammy:	We've been able to get two triangles from a square and three triangles from a pentagon. There is one condition that you need to take notice of. You can join any two points of a given polygon to form a triangle, but we don't want the situation where the lines drawn in the polygon intersect … Now, observe the polygons portrayed on the GeoGebra window and record your observations, as I drag the slider …What formula did you write for the sum of interior angles of a polygon (S) in terms of n?
S11:	We wrote $S = 180^0 (n - 2)$.
Sammy:	That is good. Very excellent work. But I saw some of you writing your answer as $S = 180° \times n - 2$. It is important to remember that it is the number of triangles in the polygon multiplied by 180^0, so $n - 2$ should be put in a bracket.

Discussion and Conclusions

This study was set out to identify the core practices of enacting effective mathematics pedagogy in a GeoGebra learning environment. This was done with the purpose of providing a response to the critique that effective mathematics pedagogy is generic and underspecified (Jacobs & Spangler, 2017). The study identified 31 core practices when the teachers used GeoGebra to enact mathematics lessons. For the purpose of summary, Table 3 shows these core practices distributed over the five central themes of effective mathematics pedagogy.

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Table 3

Core practices of effective mathematics pedagogy in a GeoGebra learning environment

Effective mathematics pedagogy	Core practices
Creating a Mathematical Setting Description: It involves the skills and knowledge of the teacher to set up the learning environment to hook and sustain the students' interest and attention throughout the mathematics lesson.	1. Preplanned lesson documents (lesson plan, worksheet, GeoGebra artefacts) 2. Set up equipment 3. Share instructional objectives with students 4. Provide clear instruction about the task on the worksheet 5. Support students to link prior knowledge to new concepts 6. Use pictures of real-life scenarios to initiate mathematical dialogue 7. Explain terminologies 8. Address existing misconceptions 9. Attention to individual learning needs
Worthwhile Mathematical Tasks Description: It involves how the objectives and activities included in the GeoGebra-based lesson engage students in exploring mathematical concepts.	1. Visualising mathematical concepts 2. Recording 3. Calculating 4. Predicting 5. Construction of new ideas
Mathematical Discussions Description: It involves creation of learning environment to facilitate classroom dialogue which emphasises on mathematical argument where conclusions are reached through agreement between students and teacher.	1. Consolidate ideas students have constructed through verbal and written response 2. Correct misconception associated with the new concept 3. Pose mathematical questions 4. Group/individual presentation 5. Revoice 6. Give students autonomy to apply concepts
Mathematical Connection Description: It involves how the teacher engages the student to use unrehearsed approach to generate multiple solutions for a problem. It includes the activities that probe students thinking and expansion of mathematical knowledge to real-life situation.	1. Repose mathematical question 2. Extend concepts learned to new contexts 3. Use GeoGebra to emphasise real-life phenomena 4. Encourage reverse thinking 5. Providing opportunity for multiple solutions 6. Risk taking

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Assessment of Students' Learning Description: It involves the creation of a learning environment where formative feedback and feedforward from the teacher and students are promoted to monitor the progress of students' learning in a specific mathematical task.	1. Review and correct student's errors 2. Use GeoGebra to provide prompt feedback 3. Remedial teaching 4. Shared ideas 5. Reflection on the solutions

Creating a Mathematical Setting

The results of the study revealed that the teachers needed to design a lesson plan, a worksheet, and a GeoGebra artefact to enhance smooth sequence of their lessons. When the teachers began to design their lessons in the GeoGebra environment, they explored and adopted more instructional strategies in anticipation of students' responses to each particular task (Stein et al., 2008). This pre-thinking about the students' expected responses strengthens not only the flow or pace of the discussion, but it also enhances the ability of the teachers to re-guide wrong answers and further provide conceptually demanding tasks to solidify and extend students' mathematical knowledge (Martin & Speer, 2009). The teachers in the current study demonstrated an ethic of care because as they designed their lessons in the GeoGebra environment, they became more pedagogically oriented by contextualising the new tool to enhance small group and whole classroom discussion.

The teachers inducted the students to their lessons through provision of explicit instructions about the tasks on the students' worksheet. They also articulated the objectives of their lessons, reviewed students' existing knowledge, used real-life scenarios to initiate communication, addressed students' existing misconceptions, and explained terminologies associated with the new concepts. Each teacher adopted at least one of these instructional strategies to induct the students into the learning. The nature of the topics they taught, the background of their students, and the teaching style and experience of the teachers accounted for variations in the way they enacted these instructional strategies. The technology played a complementary role in teachers selecting multiple real-life scenarios to support students with different learning ability and further addressed the existing mathematical errors the students brought to the classroom.

Worthwhile Mathematical Tasks

A mathematical task is worthwhile if the objectives and activities included in the lesson support students to develop procedural and conceptual fluency in mathematics (Artigue, 2002). Anthony and Walshaw (2009) talked about "thinking" and "communicating with tools" (p. 23) as important approaches for students to make sense of mathematics. In the current study, the teachers adapted GeoGebra in an exploratory approach where students made algebraic and geometric generalisation from a sequence of activities including visualising, recording, calculating, predicting and constructing new ideas. Students were challenged to make inferences based on the information they had recorded on the worksheet. For example, in Bernard's class, the affordance of the animation in GeoGebra facilitated the students to identify and to conceptualise the connection between the area of a rectangle and the curved surface of the cylinder. This offered the students a crucial step to deduce the algebraic formula for the surface area of the cylinder. As the students began to think with GeoGebra in Joshua's class, it facilitated them to grasp the key mathematical ideas required for drawing distance-time graphs when new scenarios were presented to them.

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Mathematical Discussions

One of the important indicators of effective mathematics pedagogy is the ability of the teacher to facilitate classroom dialogue that focusses on mathematical argument (Anthony & Walshaw, 2007; NCTM, 2007). As the professional development progressed, the teachers demonstrated improvement in the way they used GeoGebra and worksheets to engage students to communicate their mathematical thinking orally and in writing. The teachers demonstrated an inquiring attitude which invited the students to develop mathematical ideas. They re-voiced and edited students' responses to consolidate understanding of the concepts.

There were certain aspects of effective mathematical discussion the teacher struggled to implement throughout the professional development. The kind of discussions the teachers engaged the students in did not reflect some of the characteristics of effective mathematical discussion articulated by Stein et al., (2008) and Jacobs and Spangler (2017). For example, the students were less subjected to public scrutiny where students would engage in open debates about the solutions the groups or individual students had presented. In most cases, the teachers had predetermined answers to the questions and discussion ended abruptly when students arrived at that answers (as in the case of Martey's lesson). These teachers hardly probed students for alternative solutions. The teachers mostly invited groups they had pre-rehearsed the solution with for whole class discussion. Although this enhanced accurate presentation of mathematical facts and ideas, it prevented the students from the awareness of other possible misconceptions or alternative solutions related to the concepts they were learning.

Mathematical Connection

Another common feature across all the lessons the teachers enacted was that the instructional activities progressed in order of difficulty. The teachers created scenarios that enabled students to link newly acquired concepts to real-life phenomena. In Gideon's class for example, students predicted quadratic equations for the path of a basketball aimed at scoring and a motor rider negotiating a curve. The image of concrete representations initiated the discussion which allowed the students to appreciate the representation of parabolic graphs such as the quadratic function. Similarly, Bernard was able to challenge students to visualise, both geometrically and algebraically, the volume of the metal sheet needed to make a pipe. Hiebert and Grouws (2007) espoused that students' mathematical proficiency is facilitated through challenging tasks that offer opportunities for students to make connections among ideas, facts, and mathematical procedures. Thus, an important aspect of effective pedagogy is the provision of an important mathematical task that offers opportunities for students to extend their mathematical knowledge, thinking and skills of problem-solving.

Assessment of Students' Learning

According to Anthony and Walshaw (2009), effective teachers use a range of assessment practices to explore students' reasoning and understanding of mathematical concepts. Evidence from the current study shows that the teachers provided clear instructional objectives in their lesson plans which they used to monitor the progress of their students learning. The worksheet and dynamic objects created in the GeoGebra window played an integral part in the way the teachers assessed their students' learning. The questions the teachers included in the worksheet were structured to offer systematic and repeated exercises where students' previous knowledge was reviewed and then built on. The worksheet provided a space for students to communicate their thinking through verbal and or written responses. As teachers walked between desks, they used the

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responses the students had provided on their worksheet to sequence the instruction by providing feedback to shape the intended learning. While the worksheet provided a pen and paper approach of assessing students' learning, the teachers used GeoGebra to offer immediate feedback which generated whole class discussion. The slider and checkbox features in the GeoGebra enabled teachers to hide and unhide the solution they wanted their students to provide. The teachers unhid the solutions after the students had performed the task to enable them to compare their solutions. This helped the teachers to identify the individual students, or the groups, who were struggling with the task for remedial instructions.

Implications of the Findings

There is contention about what constitutes effective mathematics pedagogy, particularly when it comes to the use of technology in teaching and learning (Davies, 2011). This study adds to this literature by providing insights into how teachers enact effective mathematics pedagogy using GeoGebra. This study offers two key contributions to the literature of effective mathematics pedagogy. First, drawing on the earlier works of Anthony and Walshaw (2007, 2009), this study condensed the principles of effective mathematics pedagogy into five central themes: creating a mathematical setting, worthwhile mathematical tasks, mathematical discussions, mathematical connections, and assessment of students' learning. Second, it theorised 31 core practices across these themes. The study does not claim the exhaustiveness of these core practices. Rather, it provides a starting point of addressing the generic and underspecified description of effective mathematics pedagogy in the literature, particularly with regard to the use of technology in the mathematics classroom. Untangling the confounding perceptions regarding the technology professional development programme, these core practices offer potential pedagogical guidelines for supporting teachers to enact effective mathematics pedagogy with technology.

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